Controlling Canards Using Ideas From MMO

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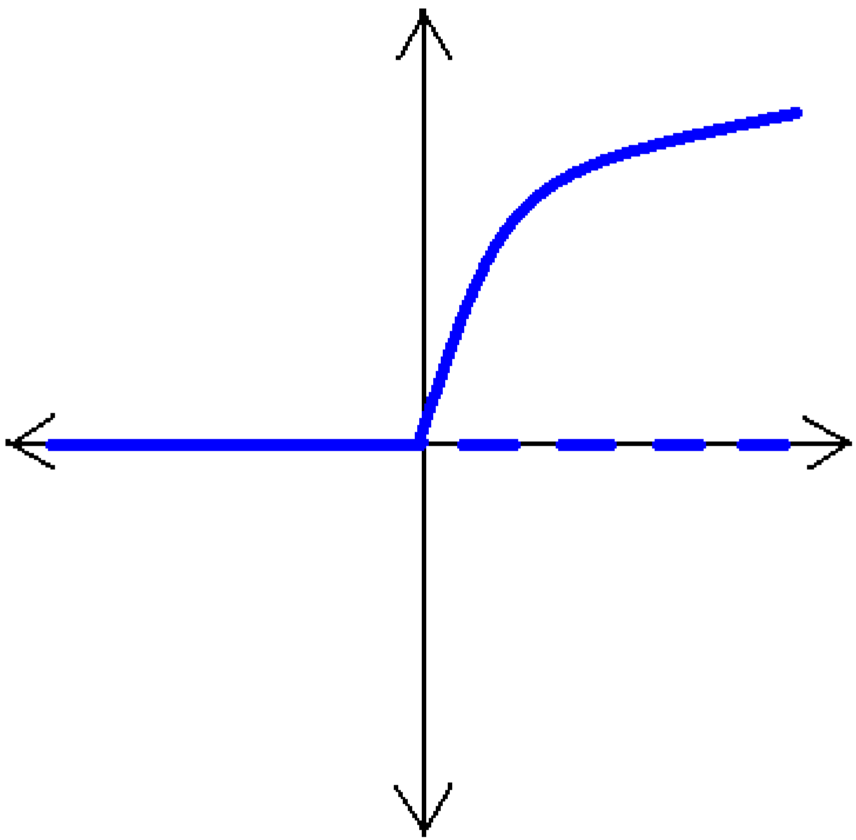
Overview

- Motivation: Hopf bifurcation control
- Canards in FitzHugh-Nagumo
- Control Circle Method
- Results
  - Best canard produced
  - Chaotic trajectories
  - Noisy MMO

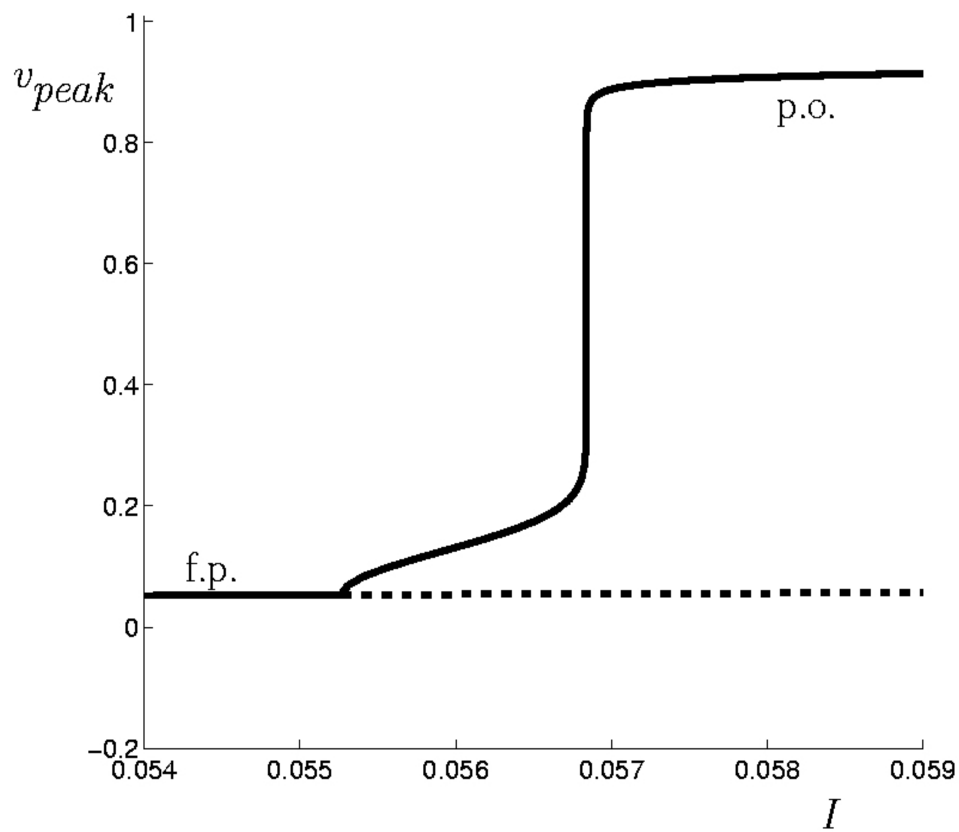
Motivation: Hopf Bifurcation Control



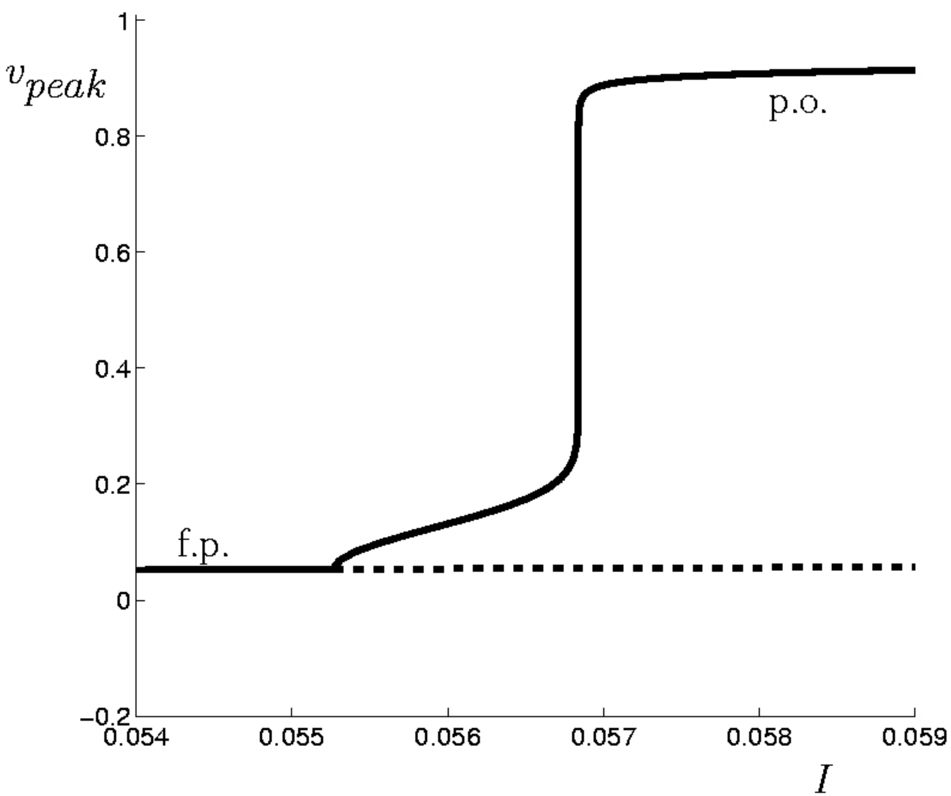
- System self-tunes a parameter to a Hopf bifurcation
  - Can be accomplished using feedback control (Moreau & Sontag, 2003)
- Systems operating at a Hopf bifurcation have:
  - Non-linear amplification
  - Noise rejection
- Crucial part of hearing

Objective:

Add feedback to a system with a canard explosion, so that the system self-tunes to the canard parameter value.



Why Canard Control?



- Canard:
  - Huge jump in p.o. size over tiny parameter change
- Could make a very sensitive sensor

FitzHugh-Nagumo Model

- Example system:

  FHN equations for neuron dynamics

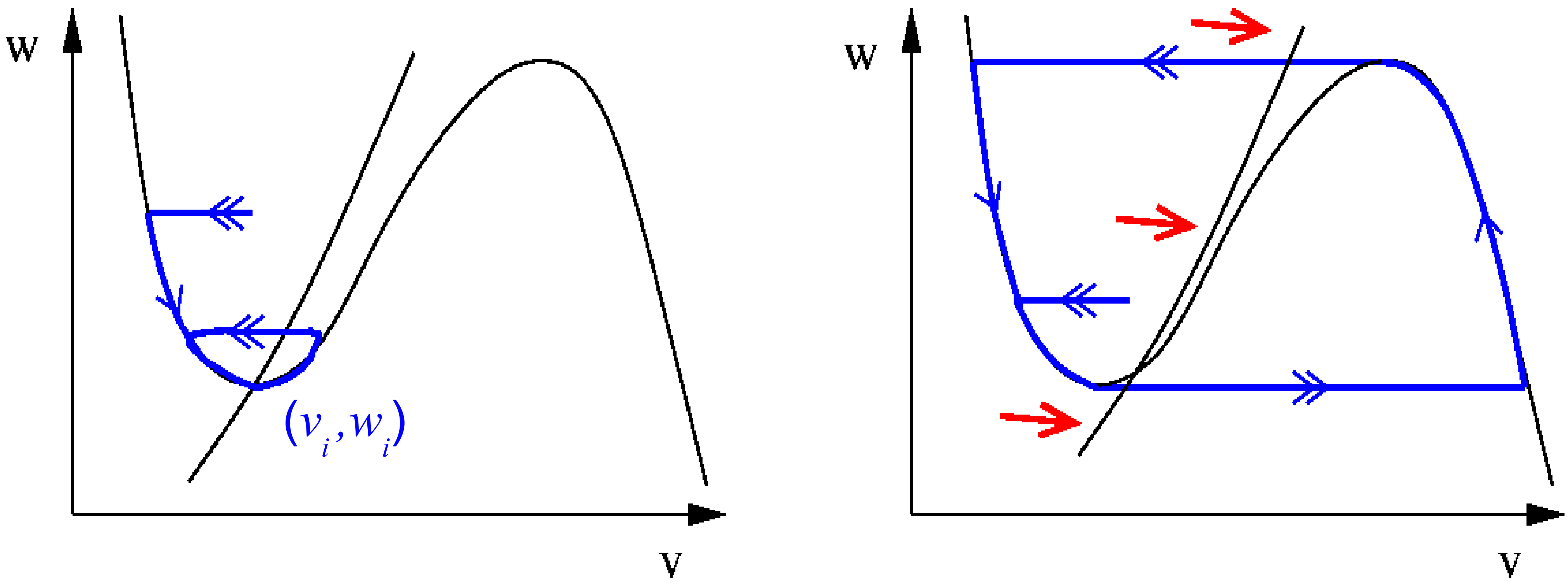
  $$\dot{v} = -w - v(v-1)(v-a) + I \equiv f(v, w; I)$$
  $$\dot{w} = \varepsilon(v - \gamma w) \equiv \varepsilon g(v, w)$$

  $\gamma = 1$, $a = 0.1$

  $\varepsilon = 0.008$

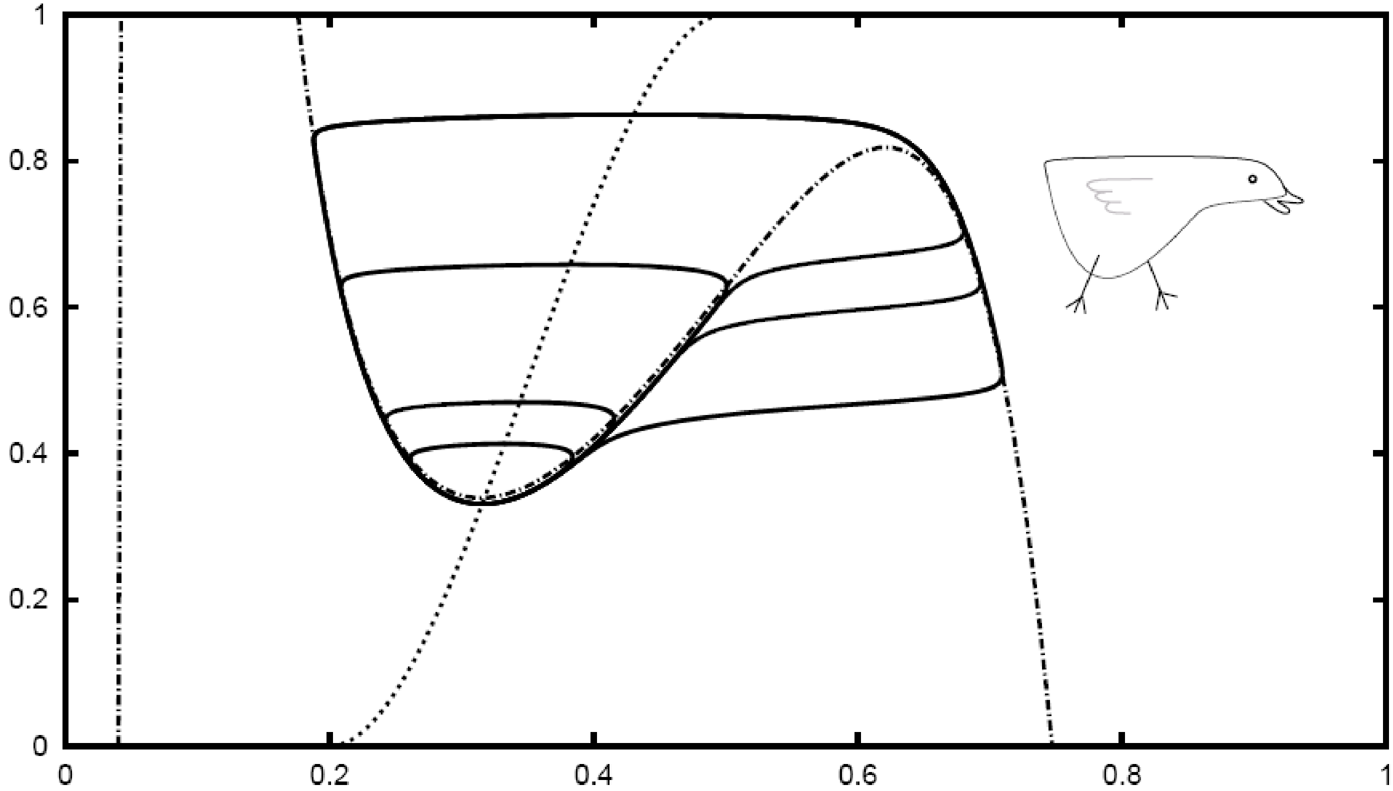
- Fast-slow system
- Nullclines occur when one of the ODEs = 0
- Parameter I controls where these intersect

Nullclines



- As I increases, nullcline shift causes periodic orbit to leave the neighborhood of (v_i, w_i)

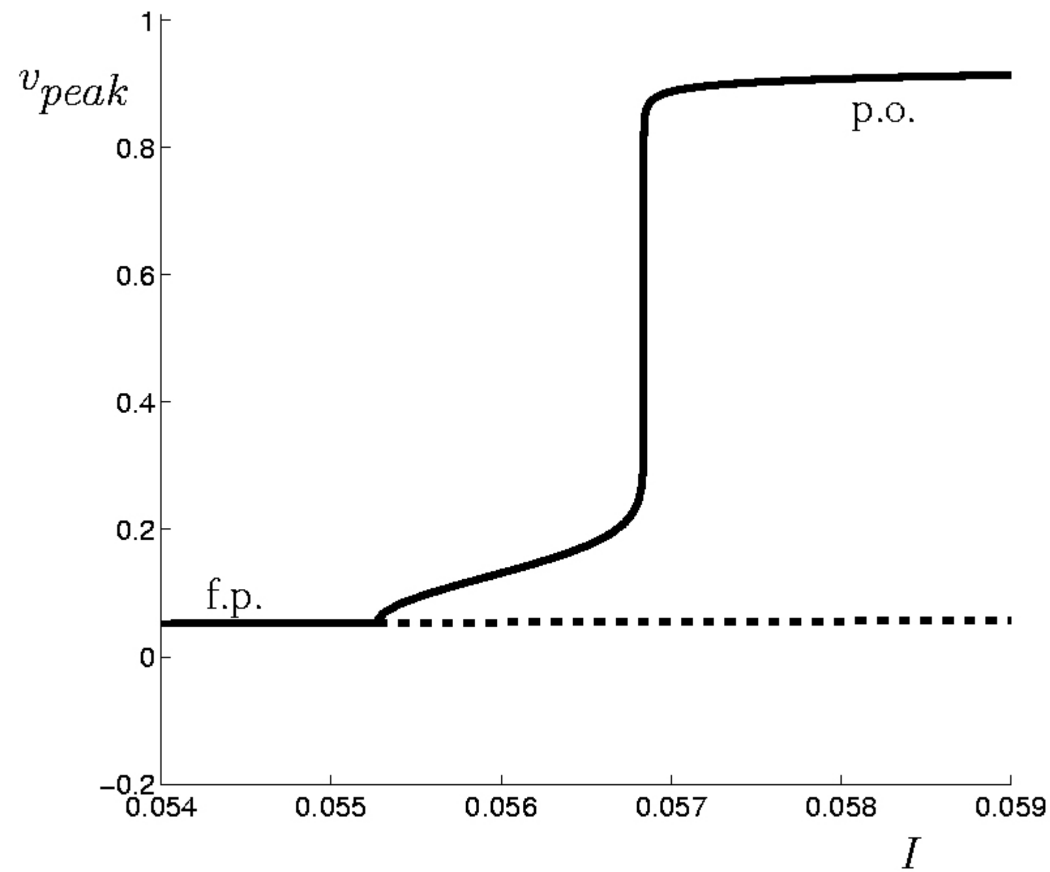
Canards



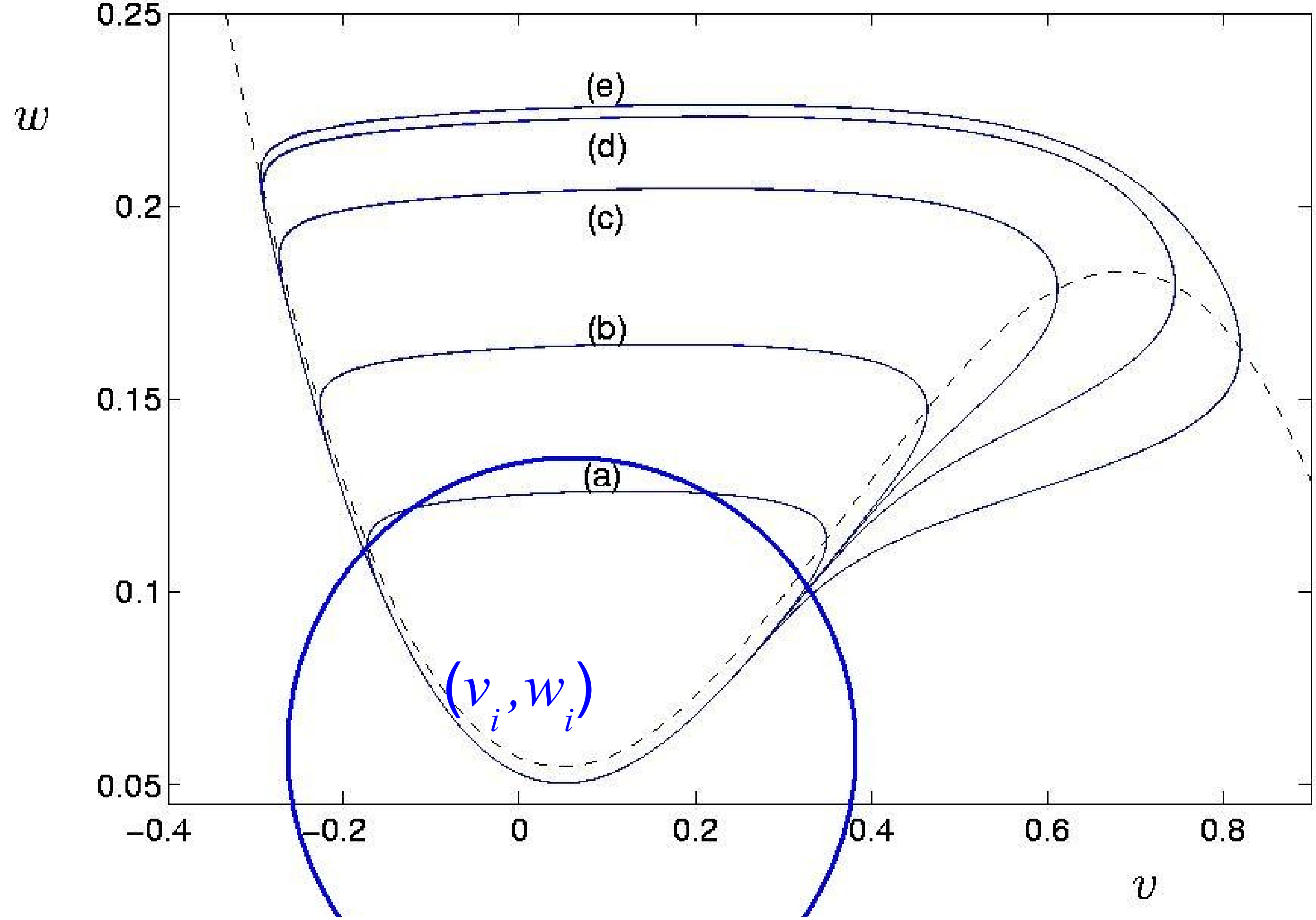
- Follow unstable manifold near middle branch of cubic v-nullcline

FHN Bifurcation Diagram

- Hopf bifurcation at I = 0.0553
- Stable p.o. grows dramatically around I = 0.0568
- Control should cause I to drift towards Canard point



Control Method



- Control circle around local minimum of v-nullcline (v_i, w_i)

Control Equations

- Controlled FHN:

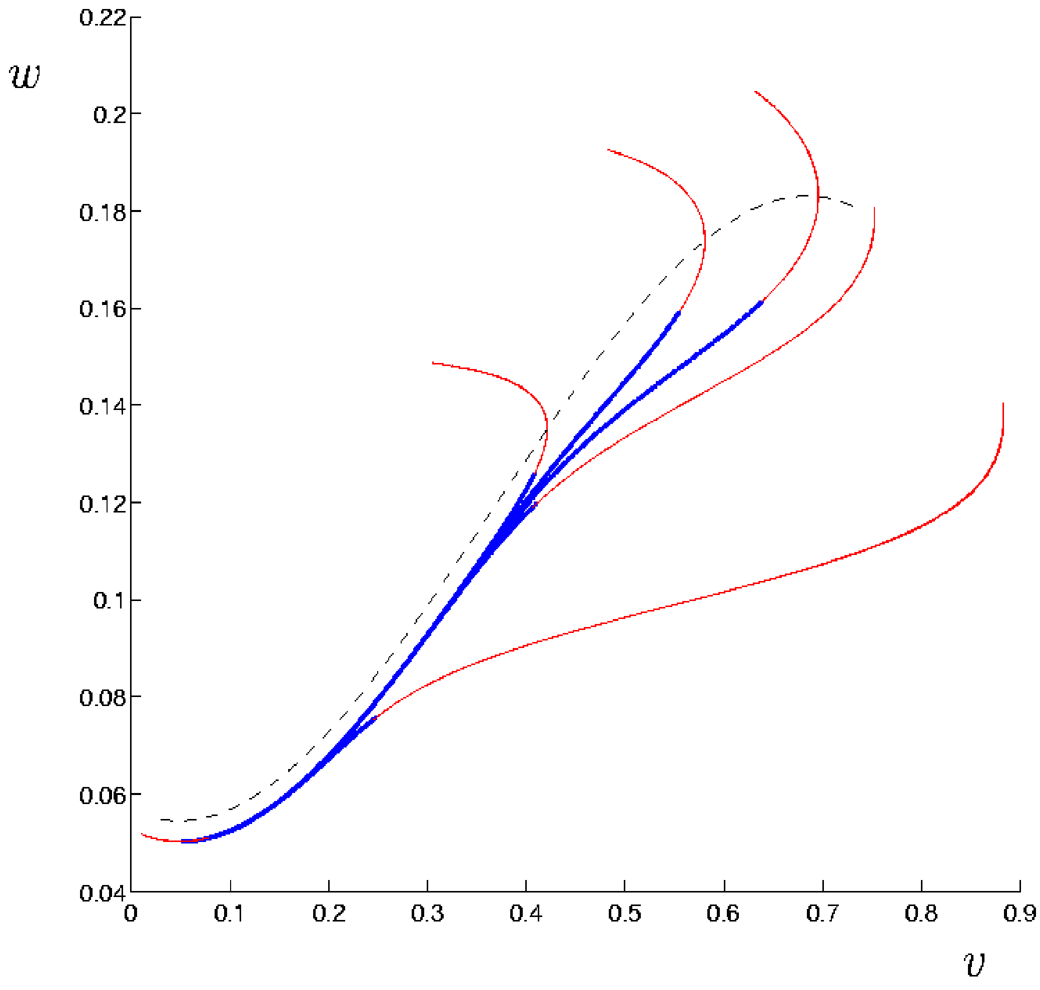
$$\dot{v} = -w - v(v-1)(v-a) + I$$

$$\dot{w} = \varepsilon(v - \gamma w)$$

$$\dot{I} = c(r_0 - r)$$

- Continuous, memoryless feedback control
  - r is the instantaneous distance from (v, w) to center of the control circle (v_i, w_i)
  - c sets control strength, r_0 sets circle radius

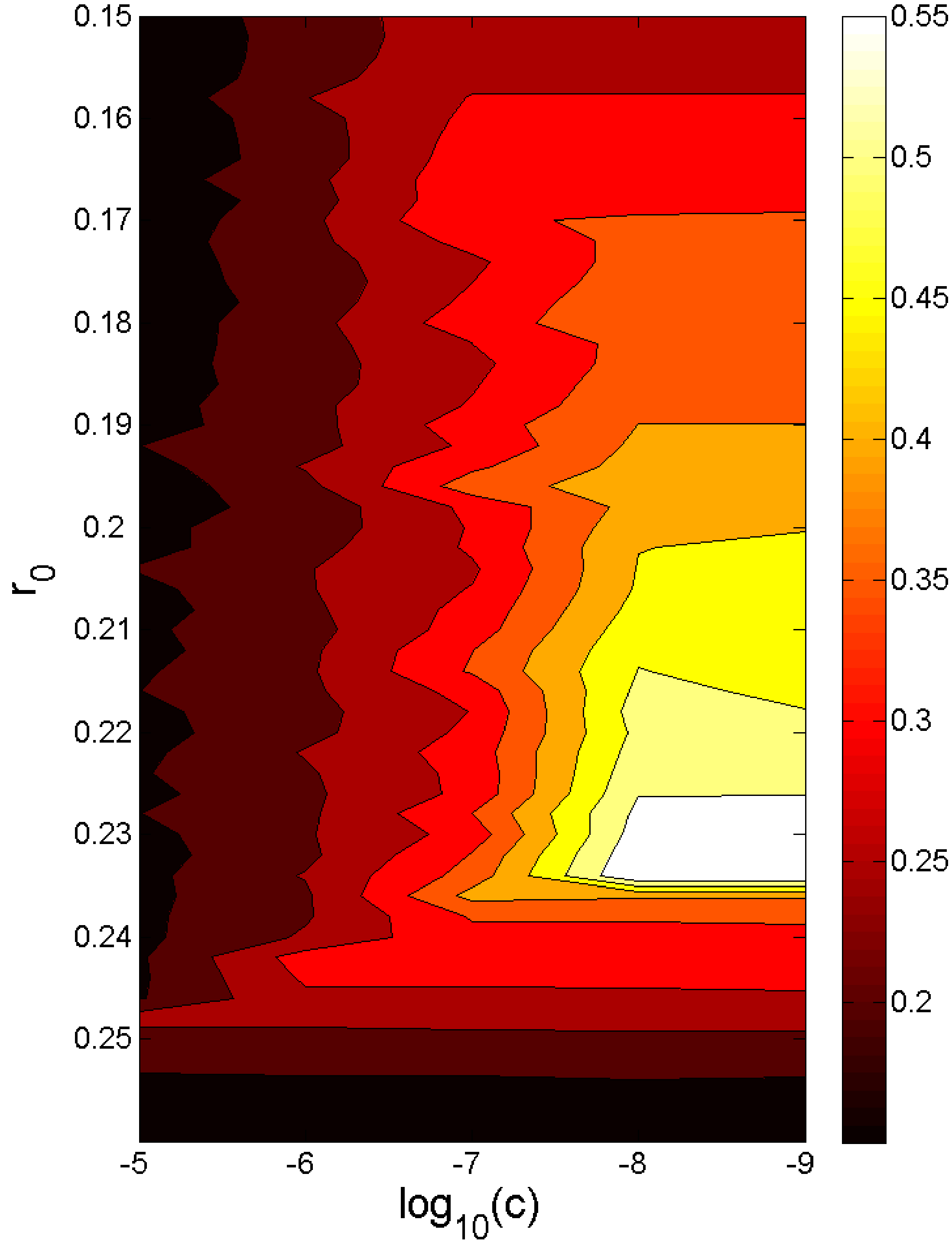
Measure Success



- How long does trajectory stay near unstable manifold
  - Manifold difficult to locate
  - But must remain close to v-nullcline
- Compare slope of trajectory and nullcline

2D Contour

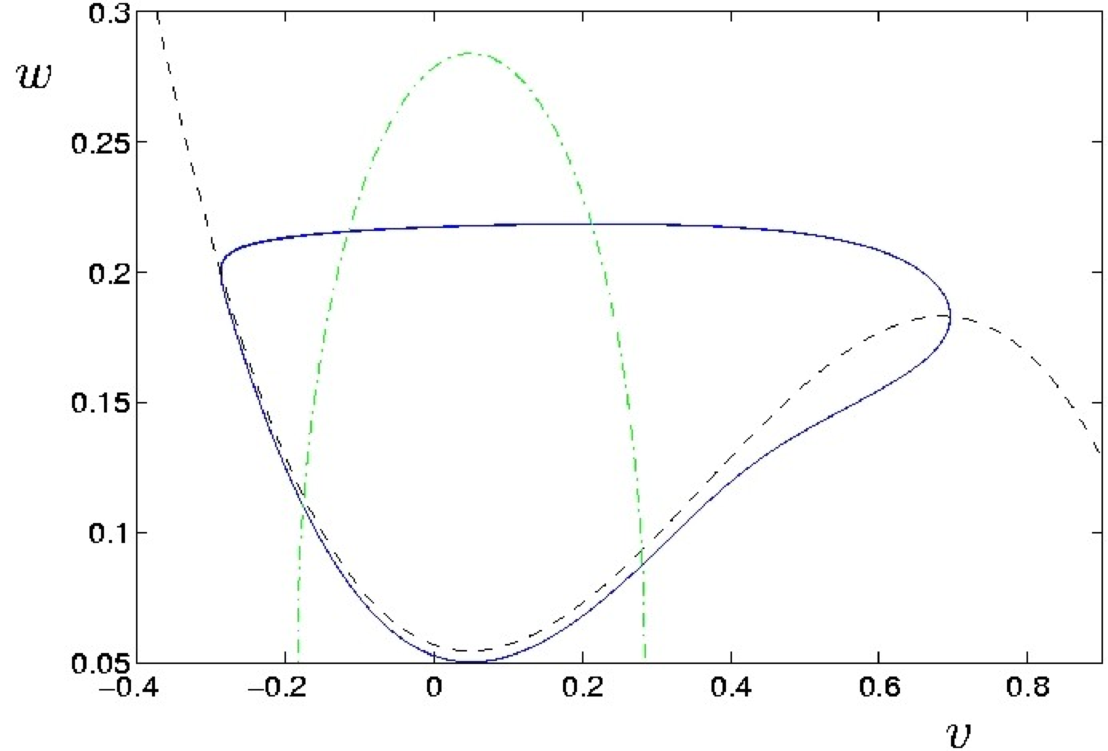
- Study of canard size over c, r_0
- Contours show averaged arclength
- Need $c = 10^{-8}$ to get full canard
- $r_0 = 0.235$ produces best result



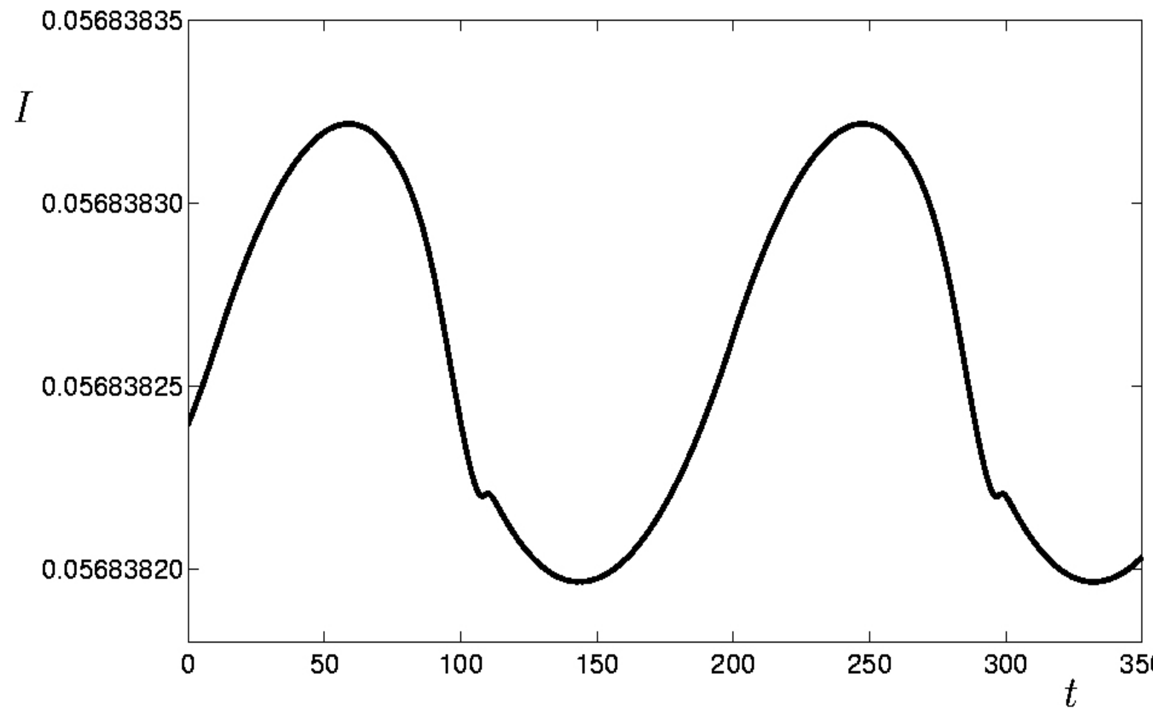
Best Canard

c = 10^{-8}

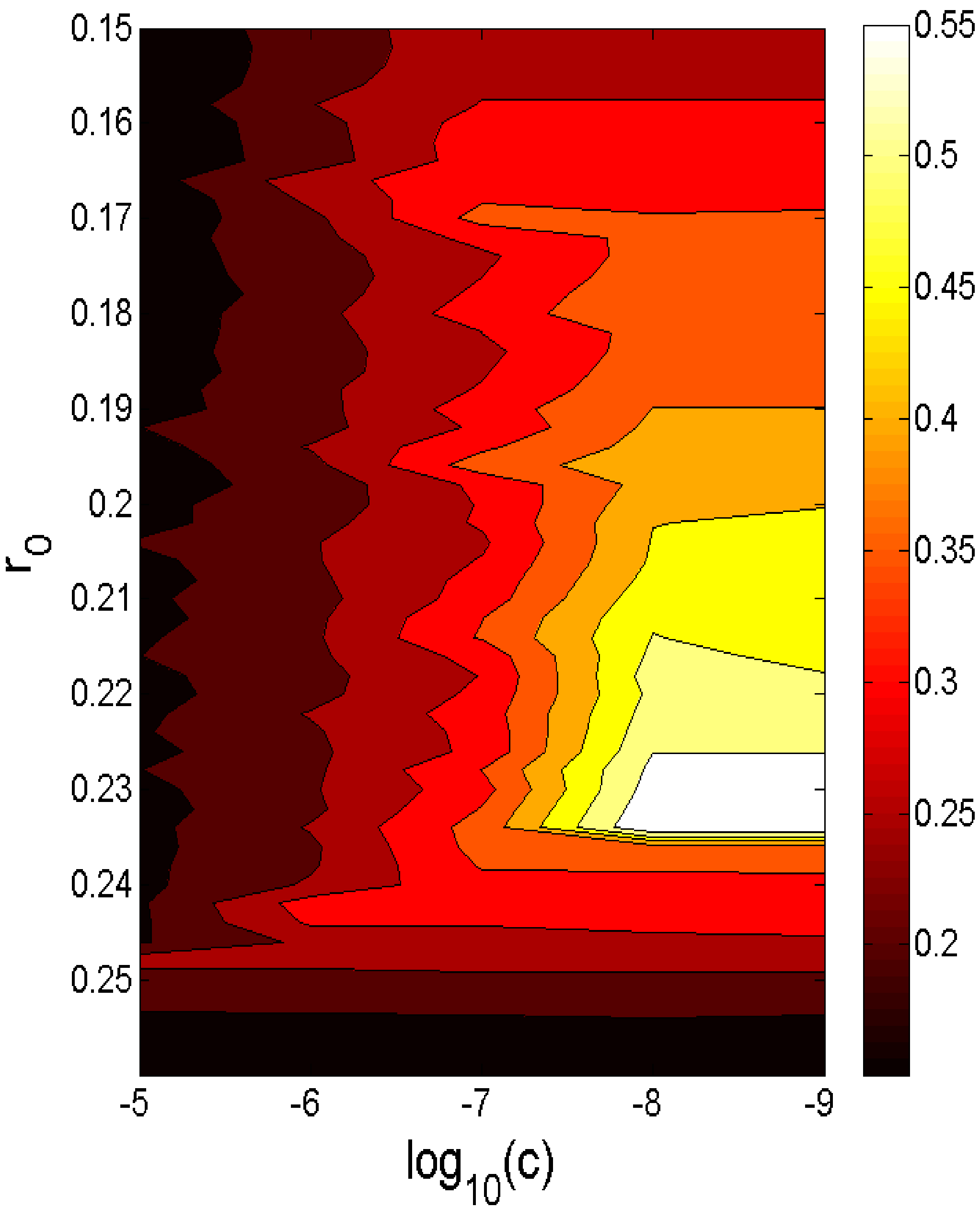
r_0 = 0.234



Time history of I for trajectory



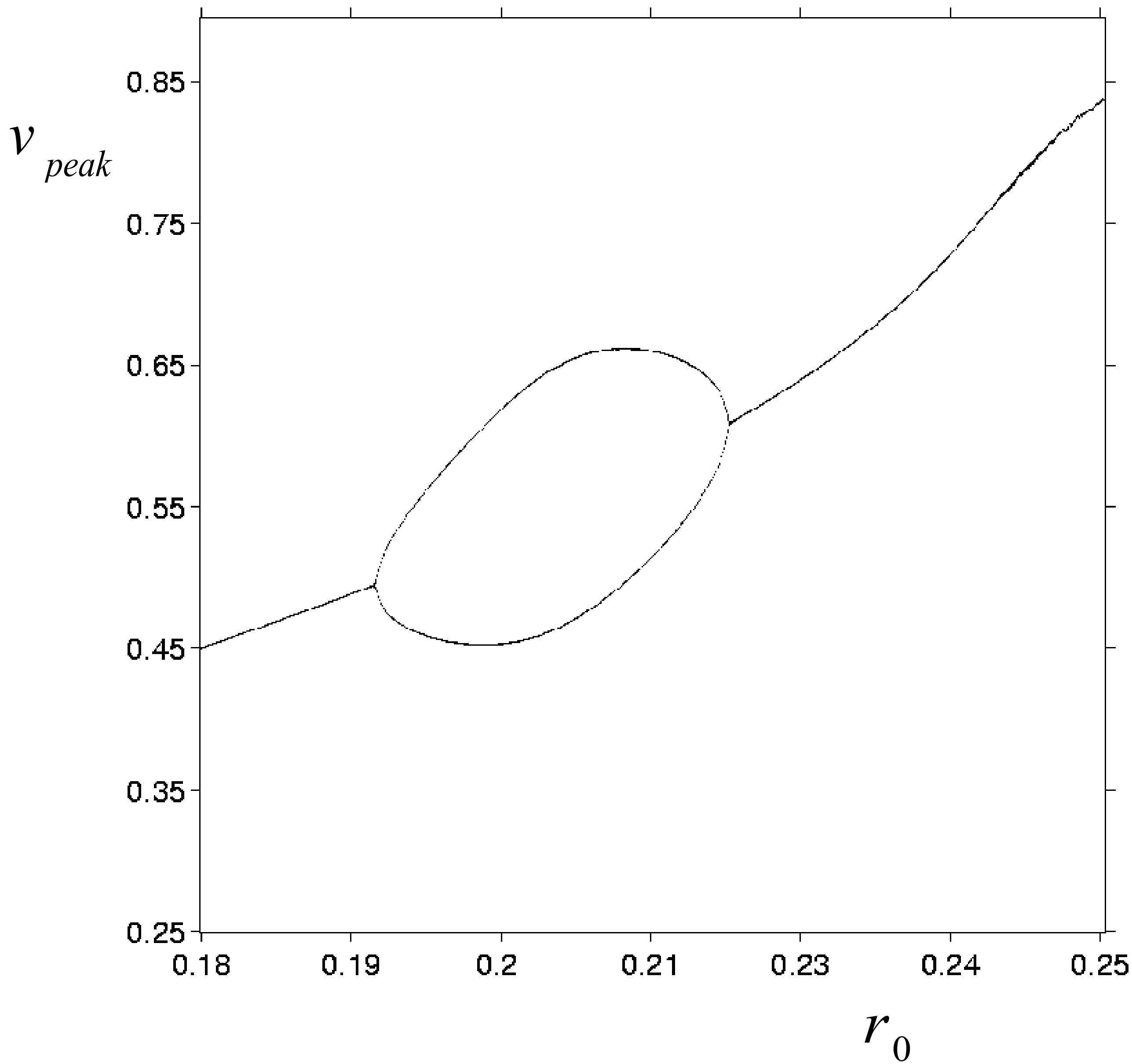
Positioning of the control circle



- Control method is fairly robust to displacements of the control circle
- Exact results change, but general picture is identical

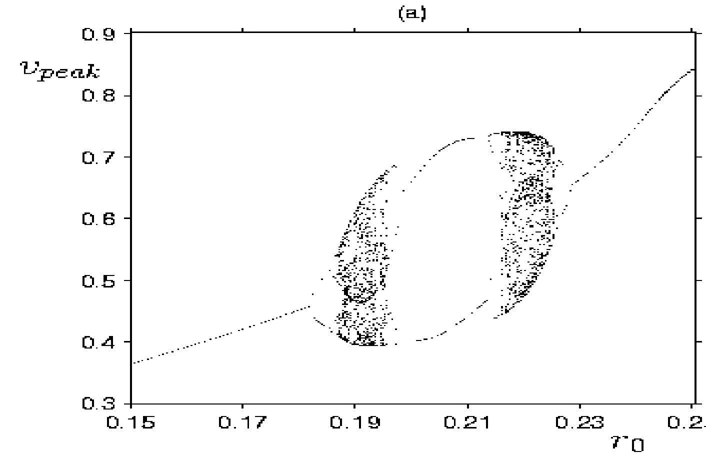
Bifurcations as r_0 changes

For $c = 10^{-8}$

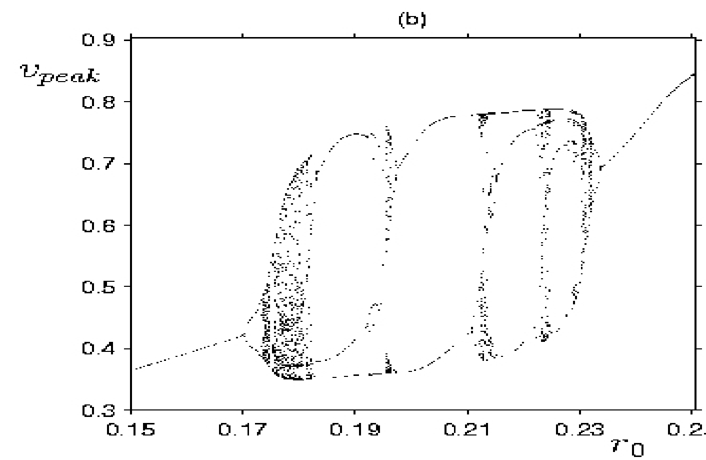


More Bifurcations

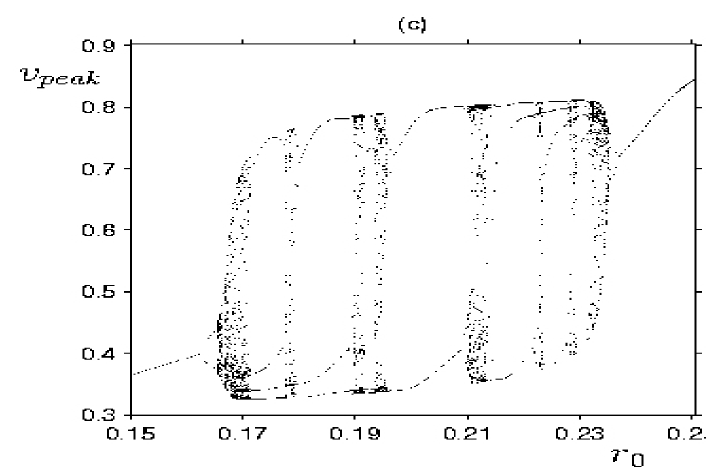
a) $c = 2 \cdot 10^{-8}$



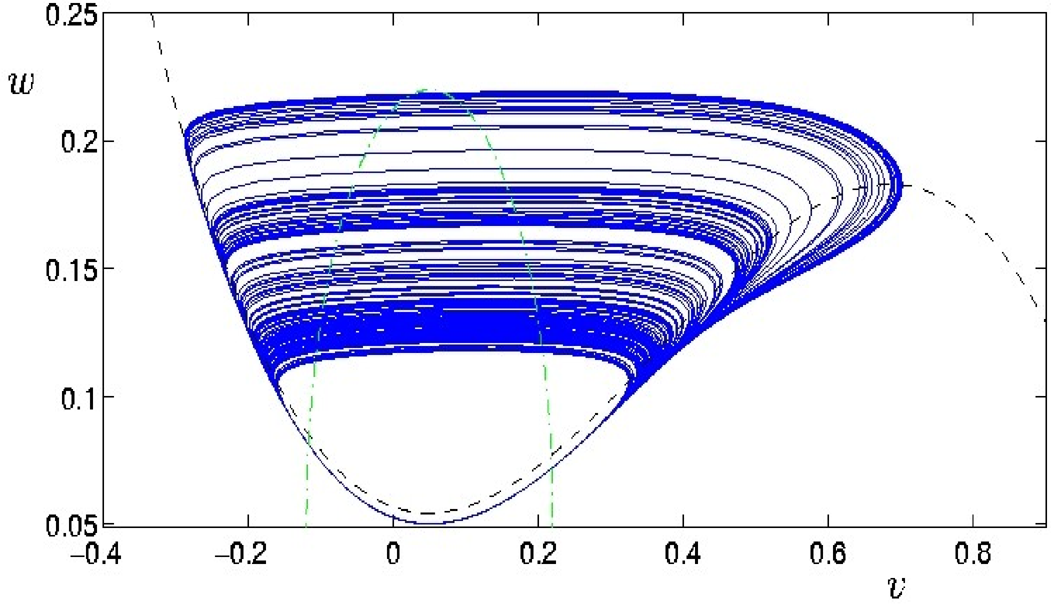
b) $c = 5 \cdot 10^{-8}$

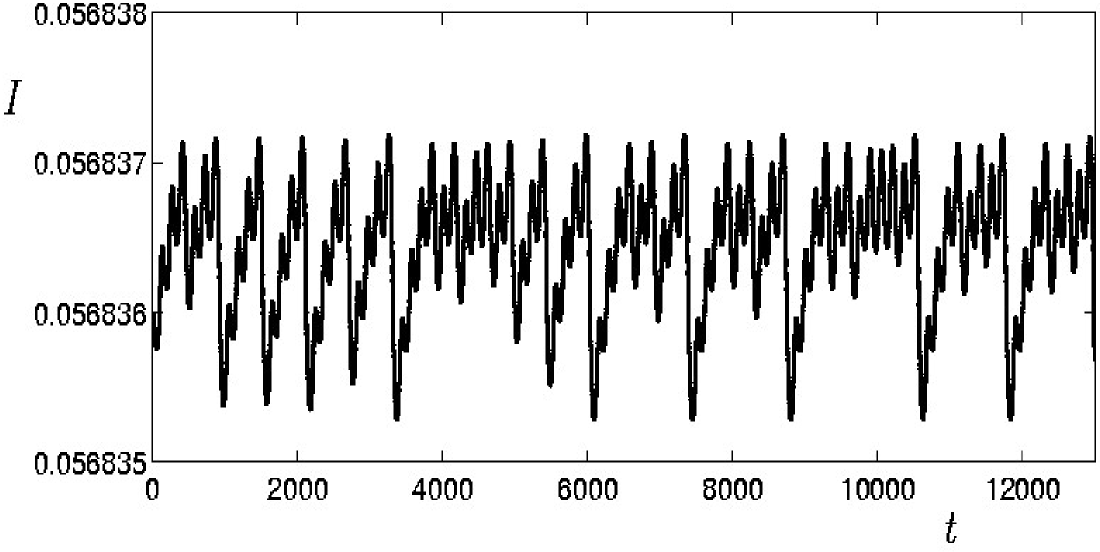


c) $c = 10^{-7}$



Chaos!



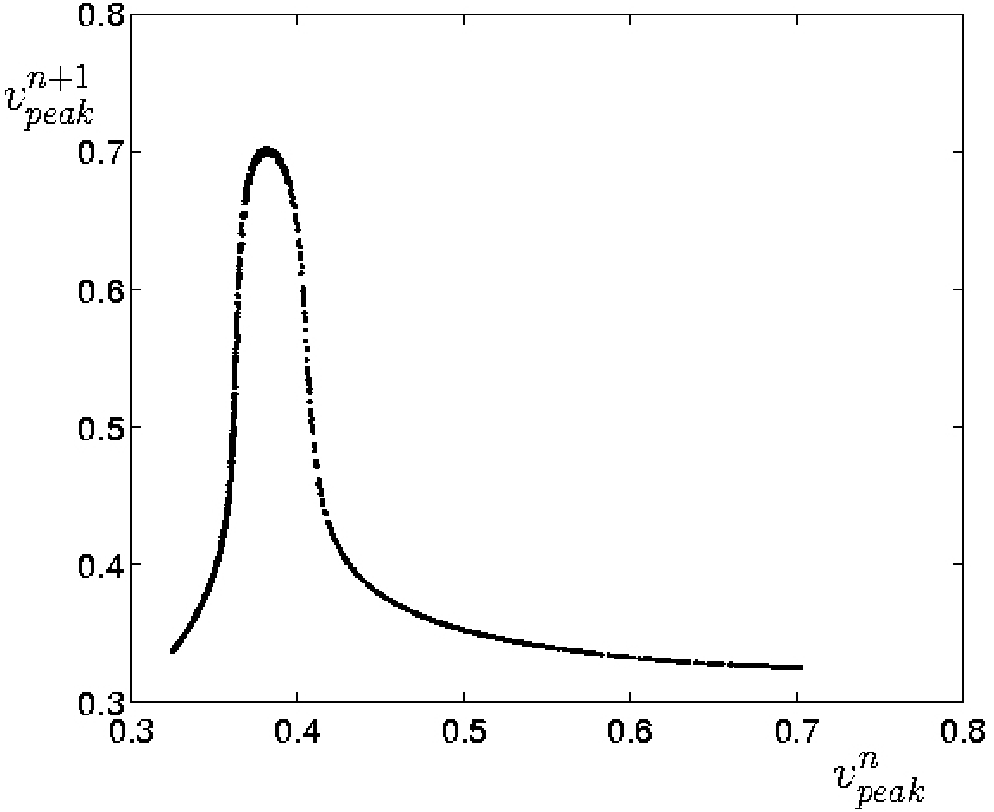


$c = 10^{-7}$

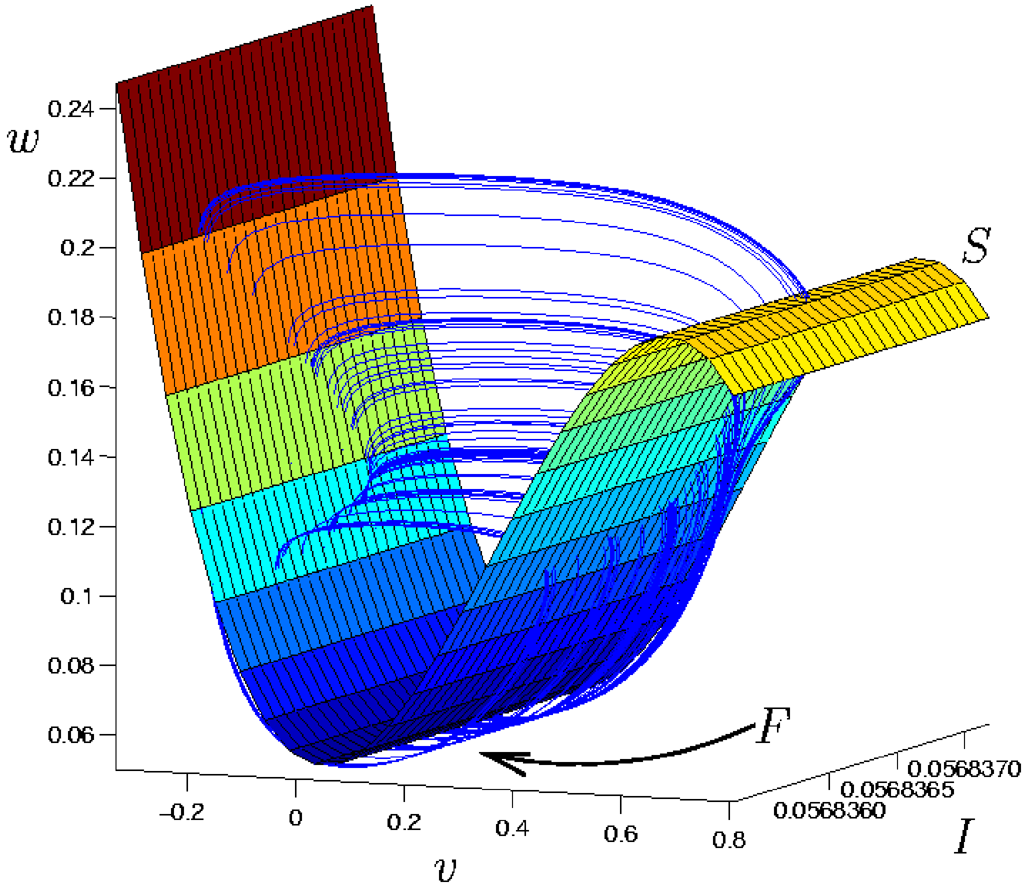
$r_0 = 0.17$

More views of chaos

Peak-to-peak map

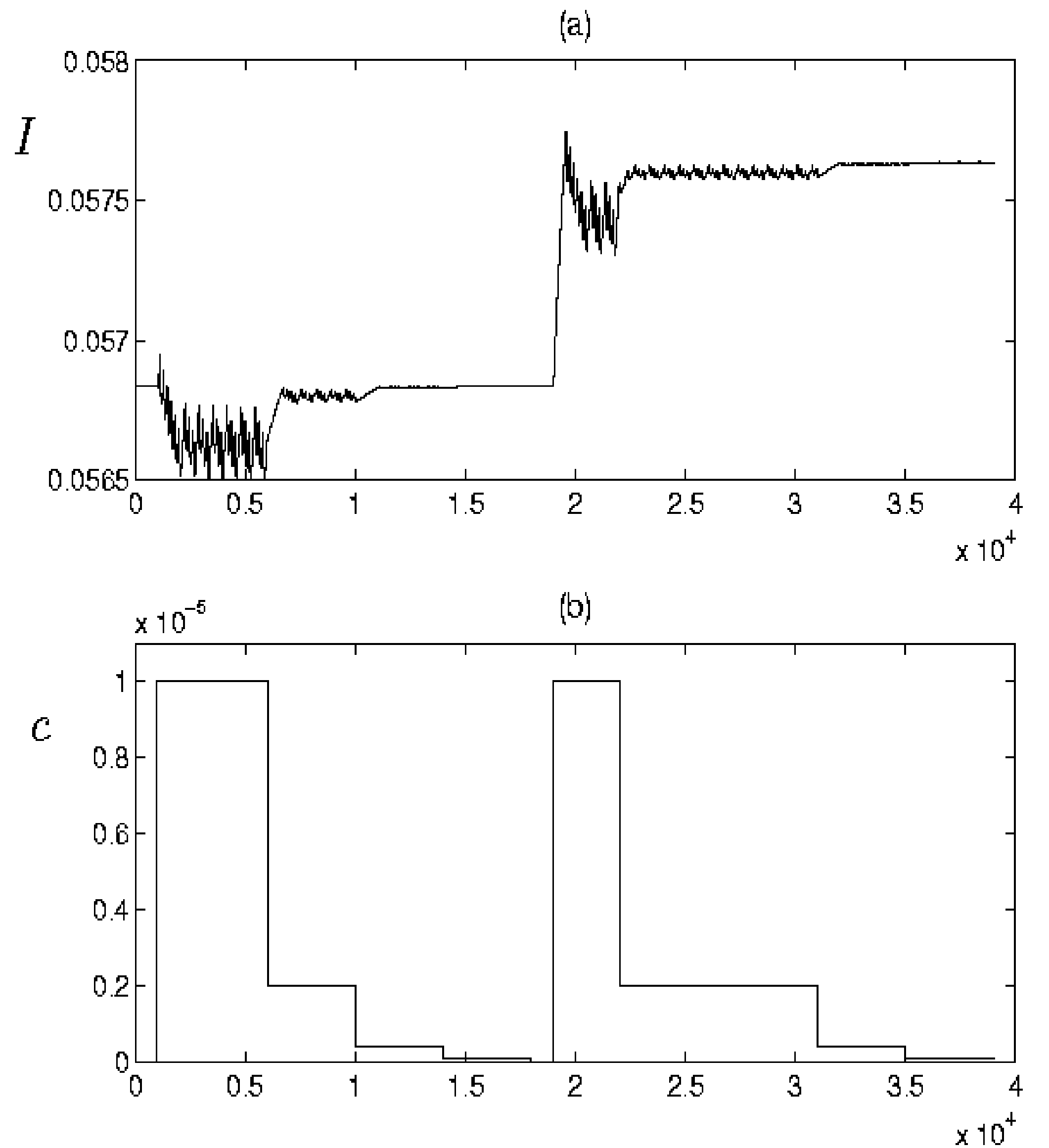


3d plot

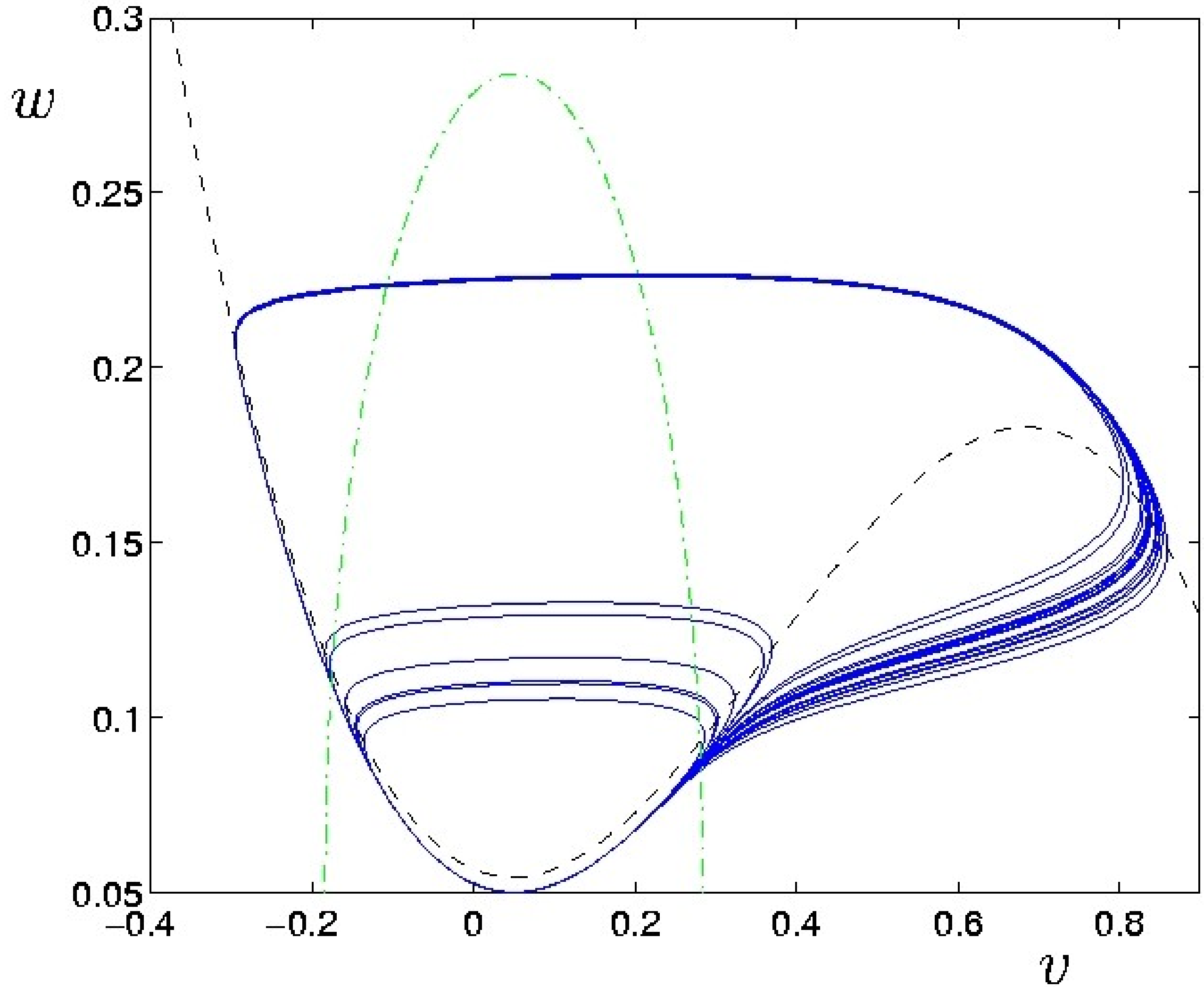


Perturbations

- Control method is robust to large but infrequent changes in system properties
- Can locate new Canard point



Continuous White Noise



Conclusions

- Can produce maximal canard trajectories
- If improperly tuned:
  - MMO
  - Chaotic MMO
- In presence of noise:
  - Noisy MMO